International Journal of Engineering, Science and Mathematics
Vol. 6 Issue 8, Decemer 2017,
ISSN: 2320-0294 Impact Factor: 6.238
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

VAGUE IDEAL OF A Γ-NEAR-RING

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Abstract

The main motivation of this paper to introduce the notion of vagu sub Γ-near-ring and vague ideal of Γ-near-ring. Based on these concepts, we analyzed some properties and results for development of theorems illustrated with examples.

Keywords:

Vague sub Γ-near-ring ;
Vague ideal of Γ-near-ring.

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1. Introduction

In 1964, Nobusawa [6] has initiated the notion of Γ-ring, which is generalization of ring. In [5, 7] was studied near-ring and gamma near-ring. The concepts of fuzzy sets and fuzzy subsets were firstly introduced by L.A.Zadeh [9] in 1965. To increase the study of vague sets, many authors have considered several extension works in fuzzy sets. W.L Gau et al [2] were the first to study the notion of vague sets. Also pointed out two important membership functions. First one is that, a true membership function and second one is false membership function. R.Biswas [1] initiated the notion of vague groups etc. In [8] S.D. Kim and H.S. Kim have studied the notion of fuzzy sub near-ring and fuzzy ideal of near-ring and P.NarsimhaSwamy [4] studied the sum of fuzzy ideal of near-ring. Y.B.Jun[3] had introduced the concepts of fuzzy ideal of gamma near-ring. In this direction, we proposed the new concepts vague sub Γ-near-ring and vague ideal of Γ-near-ring. And also, we have studied some properties and their results discussed.

2. Preliminaries

Throughout this paper N stands for Γ-near-ring unless or otherwise mentioned.

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***Definition1.**[2] A vague set A in the universe of discourse U is a pair* (t_A, f_A), where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are mappings such that $t_A(u) + f_A(u) \le 1$ for all $u \in U$. The functions t_A and f_A are called true membership function and false membership function in [0,1] respectively.

***Definition2.**[2] The interval* $[t_A(u), 1 - f_A(u)]$ *is called the vague value of u in A and it is denoted by* $V_A(u)$, *i.e.* $V_A(u) = [t_A(u), 1 - f_A(u)]$ *where A is a vague set and* $u \in U$ *is the universal of discourse or classical objects.*

***Notation1. [2]** Let* [0, 1] *denotes the family of all closed subinterval of* [0, 1]. *If* $I_1 = [a_1, b_1]$ *and* $I_2 = [a_2, b_2]$ *be two elements of* I[0, 1], *we call* $I_1 \ge I_2$ *if* $a_1 \ge a_2$ *and* $b_1 \ge b_2$ *with the order in* I[0, 1] *is a lattice with operations min or inf and max or sup given by*

$$\min\{I_1, I_2\} = [\min(a_1, a_2), \min(b_1, b_2)]$$

$$\min\{I_1, I_2\} = [\max(a_1, a_2), \max(b_1, b_2)]$$

Definition4. [8] *Let R and S be near-ring. A map* $\phi : R \to S$ *is called near-ring homomorphism if* ϕ *(x + y) =* ϕ *(x) +* ϕ *(y) and* ϕ *(xy) =* ϕ *(x)* ϕ *(y) for all x, y* $\in$ *R.*

3. Vague ideal of Γ-near-ring

Definition5. *Let A be a vague set of a* Γ *-near-ring N. Then A is called vague sub* Γ *-near-ring of N, if it satisfies the following conditions:*

(i) $V_A(x + y) \ge \min(V_A(x), V_A(Y))$,

(ii) $V_A(-x) = V_A(x)$,

(iii) $V_A(x\alpha\gamma) \ge \min(V_A(x), V_A(y))$ *for every x, y* $\in N$ *and –x is the additive inverse of x.*

Definition6. *Let A be a vague set of a* Γ *-near-ring N. Then A is said to be a vague ideal of N, if it satisfies the following conditions:*

(i) $V_A(x + y) \ge \min(V_A(x), V_A(y))$

(ii) $V_A(-x) = V_A(x)$, *-x is the additive inverse of x*

(iii) $V_A(z + x - z) \ge V_A(x)$,

(iv) $V_A(x\alpha\gamma) \ge V_A(x)$,

(v) $V_A(x\alpha(y+i) - x\alpha\gamma) \ge V_A(i)$ (Equivalently $V_A(x\alpha z - x\alpha\gamma) \ge V_A(z - y)$) *for every x,y,z,i* $\in N$ *and* $\alpha \in \Gamma$.

A is a vague right ideal of N if it satisfies (i), (ii), (iii) and (iv).

A is a vague left ideal of N if it satisfies (i), (ii), (iii) and (v).

***Note:** (a)* In the above definition the conditions (i) and (ii) together can be written as $V_A(x - y) \ge \min(V_A(x), V_A(y))$.

(b) If *A* is a vague ideal of *N*, then $V_A(x + y) = V_A(y + x)$ for every *x, y* $\in$ N.

(c) If *A* is a vague ideal of *N*, then $V_A(0) \ge V_A(x)$ for every *x* $\in$ N.

***Example1.** Let* $N_1 = \{0,1,2\}_{\oplus 3}$ *and* $\Gamma = \{\alpha, \beta\}$ *define a mapping on* N_1 *as* $N_1 \times \Gamma \times N_1 \to N_1$ *by the following tables*

⊕3	0	*1*	*2*
0	0	1	2
1	1	2	0
2	2	0	1

α	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

β	0	1	2
0	0	0	0
1	0	1	2
2	0	1	2

It is clear that x, y, z,i $\in N_1$ *and α, β* $\in \Gamma$. *Then* N_1 *is a* $\Gamma-$ *near-ring.*

A vague set $A=(t_A,\ f_A)$ *of* N_1 *defined as* $t_A : N_1 \to [0,1]$ *and* $f_A : N_1 \to [0,1]$ *by*

$$t_A(x) = \begin{cases} 0.5 & \text{if } x=0, \\ 0.5 & \text{if } x=1,2. \end{cases}$$

$$f_A(x) = \begin{cases} 0.5 & \text{if } x=0, \\ 0.5 & \text{if } x=1,2. \end{cases}$$

Then A is a vague ideal of N_1 *for every* $x, y \in N_1$.

Remark1. *Let A be a vague ideal of N, then the condition* $V_A(x\alpha z - x\alpha y) \geq V_A(z-y)$ *is equivalent to the condition* $V_A(x\alpha(y+i) - x\alpha y) \geq V_A(i)$.

Proof: Suppose that $V_A(x\alpha z - x\alpha y) \geq V_A(z-y)$

Then $V_A(x\alpha(y+i) - x\alpha y) \geq V_A(y+i-y) = V_A(i).$

Conversely, suppose that $V_A(x\alpha(y+i) - x\alpha y) \geq V_A(i)$

Then $x\alpha z - x\alpha y = x\alpha(y-y+z) - x\alpha y = x\alpha(y+i) - x\alpha y$ *where* $i = -y+z.$

$$\begin{aligned} \text{Thus } V_A(x\alpha z - x\alpha y) &= V_A(x\alpha(y+i) - x\alpha y) \\ &\geq V_A(i) \\ &= V_A(-y+z) \\ &= V_A(z-y). \end{aligned}$$

Lemma1. *Let N be a* Γ*-near-ring and A be a vague set of N satisfies the condition* $V_A(x-y) \geq \min(V_A(x), V_A(y)$ *then (i)* $V_A(0)) \geq V_A(x)$ $(ii) V_A(-x) \geq V_A(x)$.

Proof.

$$\begin{aligned} (i)\ V_A(0) &= V_A(x-x) \\ &\geq \min(V_A(x),\ V_A(-x) \\ &\geq \min(V_A(x),\ V_A(x)) \\ &= V_A(x) \ \textit{for every } x \in N \end{aligned}$$

$$\begin{aligned} (ii)\ V_A(-x) &= V_A(0-x) \\ &\geq \min(V_A(0),\ V_A(-x) \\ &\geq \min(V_A(x),\ V_A(x)) \\ &= V_A(x) \end{aligned}$$

For all $x \in N$.

Proposition1. *Let A be a vague ideal of* Γ *-near-ring N. If* $V_A(x-y)=V_A(0)$ *then* $V_A(x)=V_A(y)$

Proof. S

Suppose that $V_A(x-y)=V_A(0)$ *for all* $x,y \in N$.

$$
\begin{aligned}
then\ V_A(x) &= V_A(x-y+y) \\
&\geq \min(V_A(x-y),V_A(y)) \\
&\geq \min(V_A(0),V_A((y)) \\
&= V_A(y)
\end{aligned}
$$

Similarly, using $V_A(y-x)=V_A(x-y)=V_A(o)$, *we have* $V_A(y)\geq V_A(x)$.

Definition7. *Let A be a vague set of N, and g is a function defined on N, then the vague set h in g(N) define by* $V_h(y)=\sup_{x\in g^{-1}(y)} V_A(x)$ *for every y* $\in$ *g(N)is called the image of A under g.*

Similarly, if B is a vague set in g (N), then the vague set A=hog in N (i.e, the vague set defined as $V_A(x)=V_h(g(x))$ *(for every x in N) is called the pre-image of h under g.*

Theorem1. *A* Γ *-near-ring homomorphic pre-image of a vague left {right) ideal is a vague left {right) ideal.*

Proof. Let $\psi : N \to S$ be a Γ -near-ring homomorphism, and h be a vague left ideal of S and A be the pre-image of h under ψ and α in N.

$$
\begin{aligned}
then V_A(x-y) &= V_h(\psi(x-y)) \\
&= V_h(\psi(x)-\psi(y)) \\
&\geq \min(V_h(\psi(x),V_h(\psi(y)) \\
&= \min(V_A(x),V_A(y))
\end{aligned}
$$

and

$$
\begin{aligned}
V_A(x\alpha y) &= V_h(\psi(x\alpha y)) \\
&= V_h(\psi(x)\alpha\psi(y) \\
&= V_A(y)
\end{aligned}
$$

and

$$
\begin{aligned}
V_A(y+x-y) &= V_h(\psi(y+x-y)) \\
&= V_h(\psi(y)+\psi(x)-\psi(y)) \\
&\geq V_h(\psi(x) \\
&= V_A(x))\ \ for\ every\ x,y\in N
\end{aligned}
$$

Now suppose that h is a vague right ideal of S, then

$$
\begin{aligned}
V_A((x+i)\alpha y - x\alpha y) &= V_h(\psi(x+i)\alpha y + x\alpha y) \\
&= V_h\left((\psi(x)+\psi(i)\alpha\psi(y)+\psi(x)\alpha\psi(y)\right) \\
&\geq V_h(\psi(i)) \\
&= V_A(x)\ \ for\, every\, x,y,i\in N.
\end{aligned}
$$

Definition: We say that a vague set A in N has the sup property if, for any subset T of N, $\exists t_0 \in T$ such that

$$V_A(t_0)=\sup_{t\in T} V_A(t)$$
.

Theorem2. *A* Γ*-near-ring homomorphic Image of a vague left(right) ideal having the sup property is a vague left{right) ideal.*

*Proof.*Let $\psi : N \to S$ be a Γ-near-ring homomorphism, and A be a vague left ideal of N with the sup property and h be the image of A under ψ. Given $\psi(x), \psi(y) \in \psi(N)$ and Let $x_0 \in \psi^{-1}(\psi(x)), y_0 \in \psi^{-1}(\psi(y))$ be such that $V_A(x_0) = \sup_{t\in\psi^{-1}(\psi(x))} V_A(t),$

$V_A(y_0) = \sup_{t\in\psi^{-1}(\psi(y))} V_A(t)$ respectively. Then

$$\begin{aligned} V_h(\psi(x)-\psi(y)) &= \sup_{t\in\psi^{-1}(\psi(x)-\psi(y))} V_A(t) \\ &\geq V_A(x_0 - y_0) \\ &\geq \min(V_A(x_0), V_A(y_0)) \\ &= \min\left(\sup_{t\in\psi^{-1}(\psi(x))} V_A(t), \sup_{t\in\psi^{-1}(\psi(y))} V_A(t) \right) \\ &= \min(V_h(\psi(x)), V_h(\psi(y)) \end{aligned}$$

and

$$\begin{aligned} V_h(\psi(x)\alpha\psi(y) &= \sup_{t\in\psi^{-1}(\psi(x)\alpha\psi(y))} V_A(t) \\ &\geq V_A(x_0 \alpha y_0) \\ &\geq V_A(y_0) \\ &= \sup_{t\in\psi^{-1}(\psi(y))} V_A(t) \\ &= V_h(\psi(y)) \end{aligned}$$

$$\begin{aligned} V_h(\psi(y+x-y) &= V_h(\psi(y)+\psi(x)-\psi(y)) \\ &= \sup_{t\in\psi^{-1}(\psi(y)+\psi(x)-\psi(y))} V_A(t) \\ &\geq V_A(y_0 + x_0 - y_0) \\ &\geq V_A(x_0) \\ &= \sup_{t\in\psi^{-1}(\psi(x))} V_A(t) \\ &= V_h(\psi(x)). \end{aligned}$$

this shows that A is a vague left ideal of $\psi(N)$. Next assume that A is a vague right ideal of N. Given $\psi(i) \in \psi(N)$, *let* $i_0 \in \psi^{-1}(\psi(i)$

$$V_A(i_0) = \sup_{t\in\psi^{-1}(\psi(i))} V_A(t) \text{ then}$$

$$\begin{aligned} V_h(\psi(x+i)\alpha y - x\alpha y)) &= V_h(\psi(x)+\psi(i))\alpha\psi(y) - \psi(x)\alpha\psi(y)) \\ &\geq \sup_{t\in\psi^{-1}(\psi(x)+\psi(i))\alpha\psi(y)-\psi(x)\alpha\psi(y))} V_A(t) \\ &\geq V_A((x_0+i_0)\alpha y_0 - x_0\alpha y_0 \\ &\geq V_A(i_0) \\ &= \sup_{t\in\psi^{-1}(\psi(x))} V_A(t) \\ &= V_h(\psi(i)). \end{aligned}$$

This shows that h is a vague right ideal of ψ (N).

Acknowledgement: The authors wish to thank the anonymous revieweras for their comments and valuable suggestions.

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